

# Absolute Magnitude Calibration for Giants based on the Colour-Magnitude Diagrams of Galactic Clusters. II-Calibration with SDSS

*S. Karaali<sup>†,A,B</sup>, S. Bilir<sup>A</sup>, and E. Yaz Gökçe<sup>A</sup>*

<sup>A</sup> Istanbul University, Faculty of Sciences, Department of Astronomy and Space Sciences, 34119, Istanbul, Turkey

<sup>B</sup> Email: karsa@istanbul.edu.tr

**Abstract:** We present an absolute magnitude calibration for red giants with the colour magnitude diagrams of six Galactic clusters with different metallicities i.e. M92, M13, M3, M71, NGC 6791 and NGC 2158. The combination of the absolute magnitudes of the red giant sequences with the corresponding metallicities provides calibration for absolute magnitude estimation for red giants for a given  $(g-r)_0$  colour. The calibration is defined in the colour interval  $0.45 \leq (g-r)_0 \leq 1.30$  mag and it covers the metallicity interval  $-2.15 \leq [Fe/H] \leq +0.37$  dex. The absolute magnitude residuals obtained by the application of the procedure to another set of Galactic clusters lie in the interval  $-0.28 < \Delta M \leq +0.43$  mag. However, the range of 94% of the residuals is shorter,  $-0.1 < \Delta M \leq +0.4$  mag. The mean and the standard deviation of (all) residuals are 0.169 and 0.140 mag, respectively. The derived relations are applicable to stars older than 2 Gyr, the age of the youngest calibrating cluster.

**Keywords:** stars: distances - (stars:) giants - (Galaxy:) globular clusters: individual (M92, M13, M3, M71) - (Galaxy:) open clusters: individual (NGC 2158, NGC 6791)

## Introduction

The distance of an astronomical object plays an important role in deriving absolute magnitudes of stars and determining the three dimensional structure of the Milky Way galaxy. The distance to a star can be evaluated by trigonometric or photometric parallaxes. Trigonometric parallaxes are only available for nearby stars where *Hipparcos* (ESA 1997) is the main supplier for the data. For stars at large distances, the use of photometric parallaxes is unavoidable. In other words, the study of the Galactic structure is strictly tied to precise determination of absolute magnitudes.

Different methods can be used for absolute magnitude determination where most of them are devoted to dwarfs. The method used in the Strömgen's *uvby* -  $\beta$  (Nissen & Schuster 1991) and in the *UBV* (Laird, Carney & Latham 1988) photometry depends on the absolute magnitude offset from a standard main-sequence. In recent years the derivation of absolute magnitudes has been carried out by means of colour-absolute magnitude diagrams of some specific clusters whose metal abundances are generally adopted as the mean metal abundance of a Galactic population, such as thin, thick discs and halo. The studies of Phleps et al. (2000) and Chen et al.

(2001) can be given as examples. A slightly different approach is that of Siegel et al. (2002) where two relations, one for stars with solar-like abundances and another one for metal-poor stars were derived between  $M_R$  and the colour index  $R-I$ , where  $M_R$  is the absolute magnitude in the  $R$  filter of Johnson system. For a star of given metallicity and colour, absolute magnitude can be estimated by linear interpolation of *two* ridgelines and by means of *linear* extrapolation beyond the metal-poor ridgeline.

The most recent procedure used for absolute magnitude determination consists of finding the most likely values of the stellar parameters, given the measured atmospheric ones, and the time spent by a star in each region of the H-R diagram. In practice, researchers select the subset of isochrones with  $[M/H] \pm \Delta_{[M/H]}$ , where  $\Delta_{[M/H]}$  is the estimated error on the metallicity, for each set of derived  $T_{eff}$ ,  $\log g$  and  $[M/H]$ . Then a Gaussian weight is associated to each point of the selected isochrones, which depends on the measured atmospheric parameters and the considered errors. This criterion allows the algorithm to select only the points whose values are closed by the pipeline of the corresponding survey such as RAVE. For details of this procedure we cite the works of Breddels et al. (2010) and Zwitter et al. (2010). This procedure is based on many parameters. Hence it provides absolute magnitudes with high accuracy. Also it can be applied to both dwarf and giant stars simultaneously.

In Karaali et al. (2003), we presented a procedure for the

<sup>†</sup> Retired.

photometric parallax estimation of dwarf stars which depends on the absolute magnitude offset from the main-sequence of the Hyades cluster. Bilir et al. (2008) obtained the absolute magnitude calibrations of the thin disc main-sequence stars in the optical ( $M_V$ ) and in the near-infrared ( $M_J$ ) bands using the recent reduced Hipparcos astrometric data (van Leeuwen 2007). Bilir et al. (2009) derived a new luminosity colour relation based on trigonometric parallaxes for the thin disc main-sequence stars with Sloan Digital Sky Survey (SDSS) photometry. In Karaali et al. (2012, hereafter Paper I), we used a similar procedure for the absolute magnitude estimation of red giants by using the  $V_0, (B - V)_0$  apparent magnitude-colour diagrams of Galactic clusters with different metallicities. Here, we will estimate absolute magnitudes for red giants with  $g_0, (g - r)_0$  colour-magnitude diagrams. Thus, we will give a chance to the researchers who work with SDSS photometry (Fukugita et al. 1996) for a direct estimation of the absolute magnitudes of the red giants. The outline of the paper is as follows. We present the data in Section 2. The procedure used for calibration is given in Section 3, and Section 4 is devoted to summary and discussion.

## 2 Data

Six clusters with different metallicities, i.e. M92, M13, M3, M71, NGC 6791, and NGC 2158, were selected for our program. The  $g'$  and  $r'$  magnitudes for the first five clusters were taken from Clem, Vanden Berg & Stetson (2008). Clem, Vanden Berg & Stetson observed the clusters in the  $u'g'r'i'z'$  passbands with the MegaCam wide-field imager on the Canada-France-Hawaii Telescope. Whereas the  $g$  and  $r$  magnitudes for the cluster NGC 2158 were provided by the observation of the cluster on instrumental  $ugriz$  passbands (Smolinski et al. 2011). The two sets of passbands are very similar, but not quite identical. We derived the following equations by the transformations of Rider et al. (2004) and transformed the  $g'$  and  $g' - r'$  data of Clem, Vanden Berg & Stetson (2008) to the  $g$  and  $g - r$  data. Thus, we obtained a homogeneous set of data for absolute magnitude calibration.

$$\begin{aligned} g &= g' + 0.060[(g' - r') - 0.53], \\ g - r &= 1.060(g' - r') - 0.035(r' - i') - 0.024. \end{aligned} \quad (1)$$

The range of the metallicity of the clusters given in iron abundance is  $-2.15 \leq [Fe/H] \leq +0.37$  dex. The  $(g - M_g)_0$  true distance modulus,  $E(B - V)$  colour excess, and  $[Fe/H]$  iron abundance for M92, M13, M3, M71, NGC 6791 are taken from the authors given in second order of the reference list in Table 1, whereas the ones for NGC 2158 are those of

Table 1: Data for the clusters used in our work.

Cluster	$E(B - V)$	$(g - M_g)_0$	$[Fe/H]$	Ref.
	(mag)	(mag)	(dex)	
M92	0.025	14.72	-2.15	(1), (2)
M13	0.020	14.38	-1.41	(1), (2)
M3	0.010	15.04	-1.50	(1), (3)
M71	0.280	12.83	-0.78	(1), (4)
NGC6791	0.100	12.94	0.37	(1), (5)
NGC2158	0.440	12.80	-0.25	(6)

(1) Clem, Vanden Berg & Stetson (2008), (2) Gratton et al. (1997),  
(3) Harris (1996, 2010), (4) Hodder et al. (1992), (5)  
Sandage, Lubin & VandenBerg (2003), (6) Smolinski et al. (2011).

Smolinski et al. (2011). The  $g$  and  $g - r$  data are presented in Table 2. We adopted  $R = A_V/E(B - V) = 3.1$  to convert the colour excess to the extinction. Although different numerical values appeared in the literature for specific regions of our Galaxy, a single value is applicable everywhere. Then, we used the equations  $A_g/A_V = 1.199$  and  $A_r/A_V = 0.858$ ,  $A_i/A_V = 0.639$  of Fan (1999) to evaluate the total extinctions in  $A_g$ ,  $A_r$  and  $A_i$ . Then, the equation for the selective extinction in *SDSS* is  $E(g - r)/A_V = 0.341$ .

The  $u'g'r'i'z'$  magnitudes for the clusters in Clem, Vanden Berg & Stetson (2008) were given in ridge-lines. We plotted the transformed  $g_0, (g - r)_0$  sequences on a diagram for each cluster and identified the giants by means of their positions in the diagram. Whereas, the fiducial red giant sequence of the cluster NGC 2158 given in Table 2, supplied by binning the transformed  $g$  magnitudes and  $g - r$  colours of 54 red giants in Smolinski et al. (2011). We, then fitted the fiducial sequence of giants to high degree polynomials. A fourth degree polynomial was sufficient for the clusters NGC 6791 and NGC 2158, whereas a fifth degree polynomial was necessary for a good correlation coefficient for the clusters M92, M13, M3, M71, for a good correlation coefficient. The calibration of  $g_0$  is as follows:

$$g_0 = \sum_{i=0}^5 a_i (g - r)_0^i. \quad (2)$$

The numerical values of the coefficients  $a_i$  ( $i = 0, 1, 2, 3, 4, 5$ ) are given in Table 3 and the corresponding diagrams are presented in Fig. 1. The  $(g - r)_0$ -interval in the second line of the table denotes the range of  $(g - r)_0$  available for each cluster.

## 3 The Procedure

### 3.1 Absolute Magnitude as a Function of Metallicity

The procedure consists of a slight modification of the procedure in Paper I. There, we calibrated the absolute magnitude

Table 2: Original  $g'$ ,  $g' - r'$ ,  $r' - i'$  and the transformed  $g_0$ ,  $(g - r)_0$  data for the clusters M92, M13, M3, M71 and NGC 6791. The  $g$  magnitudes and  $g - r$  colours for the cluster NGC 2158 are original.

$g' - r'$	$g'$	$r' - i'$	$g'_0$	$(g' - r')_0$	$(r' - i')_0$	$(g - r)_0$	$g_0$	$g' - r'$	$g'$	$r' - i'$	$g'_0$	$(g' - r')_0$	$(r' - i')_0$	$(g - r)_0$	$g_0$
M92								M3 (cont.)							
1.150	12.650	0.508	12.557	1.124	0.491	1.150	12.593	0.487	17.987	0.192	17.950	0.476	0.185	0.475	17.947
0.952	12.952	0.411	12.859	0.926	0.394	0.943	12.883	0.479	18.179	0.189	18.142	0.468	0.182	0.466	18.138
0.835	13.335	0.361	13.242	0.809	0.344	0.821	13.259	0.467	18.367	0.184	18.330	0.456	0.177	0.454	18.325
0.746	13.746	0.330	13.653	0.720	0.313	0.728	13.664	M71							
0.684	14.184	0.300	14.091	0.658	0.283	0.663	14.099	1.547	13.047	0.854	12.006	1.251	0.664	1.279	12.050
0.634	14.634	0.276	14.541	0.608	0.259	0.611	14.546	1.344	13.344	0.640	12.303	1.048	0.450	1.071	12.334
0.593	15.093	0.255	15.000	0.567	0.238	0.568	15.002	1.203	13.703	0.566	12.662	0.907	0.376	0.924	12.685
0.554	15.554	0.238	15.461	0.528	0.221	0.527	15.461	1.102	14.102	0.511	13.061	0.806	0.321	0.819	13.078
0.523	16.023	0.222	15.930	0.497	0.205	0.495	15.928	1.019	14.519	0.464	13.478	0.723	0.274	0.733	13.490
0.497	16.497	0.210	16.404	0.471	0.193	0.468	16.401	0.953	14.953	0.439	13.912	0.657	0.249	0.664	13.920
0.476	16.976	0.200	16.883	0.450	0.183	0.446	16.878	0.907	15.407	0.423	14.366	0.611	0.233	0.616	14.371
0.458	17.458	0.189	17.365	0.432	0.172	0.427	17.359	0.871	15.871	0.412	14.830	0.575	0.222	0.578	14.833
0.448	17.648	0.184	17.555	0.422	0.167	0.417	17.549	0.837	16.337	0.405	15.296	0.541	0.215	0.542	15.297
M13								0.813	16.813	0.399	15.772	0.517	0.209	0.517	15.771
1.330	12.630	0.540	12.556	1.309	0.526	1.345	12.602	0.795	17.295	0.394	16.254	0.499	0.204	0.498	16.252
1.071	12.871	0.460	12.797	1.050	0.446	1.073	12.828	0.789	17.489	0.391	16.448	0.493	0.201	0.492	16.446
0.933	13.233	0.398	13.159	0.912	0.384	0.929	13.182	0.781	17.681	0.381	16.640	0.485	0.191	0.483	16.638
0.824	13.624	0.353	13.550	0.803	0.339	0.815	13.566	NGC 6791							
0.750	14.050	0.320	13.976	0.729	0.306	0.738	13.988	1.414	14.514	0.730	14.142	1.308	0.662	1.340	14.189
0.694	14.494	0.292	14.420	0.673	0.278	0.679	14.428	1.298	14.898	0.543	14.526	1.192	0.475	1.223	14.566
0.647	14.947	0.268	14.873	0.626	0.254	0.631	14.878	1.214	15.314	0.468	14.942	1.108	0.400	1.137	14.977
0.607	15.407	0.250	15.333	0.586	0.236	0.589	15.336	1.145	15.745	0.424	15.373	1.039	0.356	1.065	15.404
0.570	15.870	0.235	15.796	0.549	0.221	0.550	15.797	1.086	16.186	0.389	15.814	0.980	0.321	1.004	15.841
0.538	16.338	0.224	16.264	0.517	0.210	0.517	16.263	1.040	16.640	0.363	16.268	0.934	0.295	0.956	16.293
0.509	16.809	0.214	16.735	0.488	0.200	0.486	16.732	1.006	17.106	0.348	16.734	0.900	0.280	0.921	16.757
0.486	17.286	0.206	17.212	0.465	0.192	0.462	17.208	0.977	17.577	0.341	17.205	0.871	0.273	0.890	17.226
0.474	17.574	0.200	17.500	0.453	0.186	0.450	17.495	0.965	17.765	0.339	17.393	0.859	0.271	0.877	17.413
0.461	17.761	0.195	17.687	0.440	0.181	0.436	17.681	0.946	17.946	0.335	17.574	0.840	0.267	0.857	17.593
M3								NGC 2158							
1.300	13.300	0.580	13.263	1.289	0.573	1.323	13.308	$g - r$	$g$	$(g - r)_0$	$g_0$	—	—	—	—
1.058	13.558	0.446	13.521	1.047	0.439	1.071	13.552	0.553	15.739	0.553	15.739	—	—	—	—
0.918	13.918	0.383	13.881	0.907	0.376	0.925	13.903	0.614	15.294	0.614	15.294	—	—	—	—
0.810	14.310	0.338	14.273	0.799	0.331	0.812	14.289	0.708	14.777	0.708	14.777	—	—	—	—
0.733	14.733	0.305	14.696	0.722	0.298	0.731	14.707	0.741	14.508	0.741	14.508	—	—	—	—
0.675	15.175	0.281	15.138	0.664	0.274	0.671	15.146	0.763	14.345	0.783	14.297	—	—	—	—
0.626	15.626	0.261	15.589	0.615	0.254	0.619	15.594	0.782	14.265	0.852	13.870	—	—	—	—
0.584	16.084	0.245	16.047	0.573	0.238	0.575	16.049	0.803	14.296	0.965	13.687	—	—	—	—
0.555	16.555	0.230	16.518	0.544	0.223	0.545	16.519	0.852	13.870	—	—	—	—	—	—
0.528	17.028	0.215	16.991	0.517	0.208	0.517	16.990	0.965	13.687	—	—	—	—	—	—
0.507	17.507	0.203	17.470	0.496	0.196	0.495	17.468	—	—	—	—	—	—	—	—

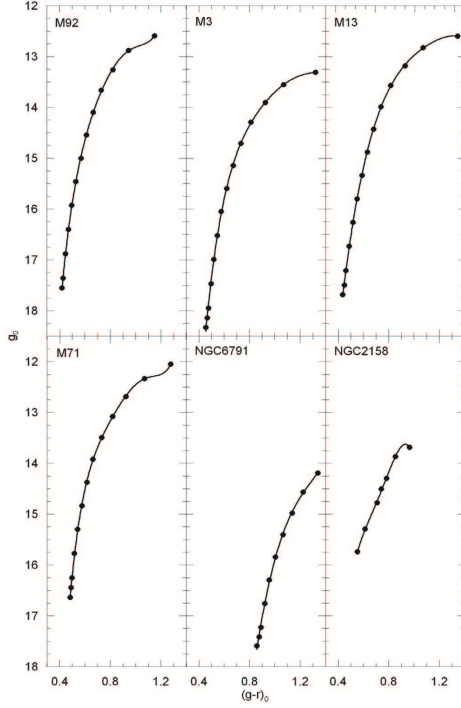


Figure 1:  $g_0, (g-r)_0$  colour-apparent magnitude diagrams for six Galactic clusters used for the absolute magnitude calibration.

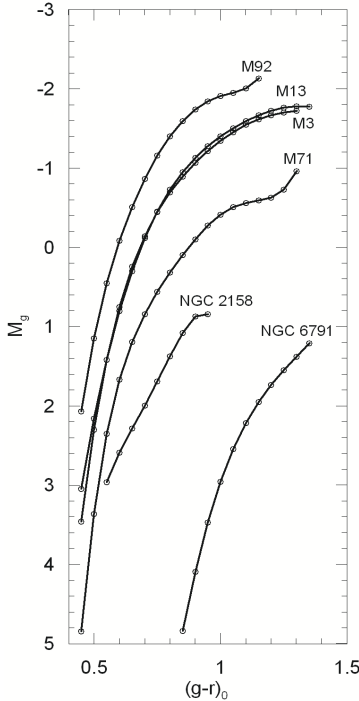


Figure 2:  $M_g, (g-r)_0$  colour-absolute magnitude diagrams for six clusters used for the absolute magnitude calibration.

Table 3: Numerical values of the coefficients  $a_i$  ( $i=0, 1, 2, 3, 4, 5$ ).

Cluster	M92	M13	M3	M71	NGC 6791	NGC 2158
$(g-r)_0$ interval	[0.42-1.15]	[0.44-1.35]	[0.45-1.32]	[0.48-1.28]	[0.86-1.34]	[0.55-0.96]
$a_5$	-107.410	1.2115	-30.463	-133.31	—	—
$a_4$	442.770	5.9697	161.410	617.40	-3.2426	201.770
$a_3$	-729.420	-41.8770	-340.950	-1132.00	-7.3559	-581.310
$a_2$	609.900	81.6260	362.750	1033.00	60.9390	624.230
$a_1$	-266.560	-69.8160	-198.370	-475.38	-96.0120	-302.130
$a_0$	63.533	35.8660	59.319	102.71	61.5710	71.356

offsets from the fiducial red giant sequence of a standard cluster (M5) for a given colour index as a function of metallicity offsets. Whereas, here we calibrated the absolute magnitudes directly to metallicities for a given  $(g-r)_0$  colour. Thus, one does not need to calculate an absolute magnitude offset from a standard cluster and then to add it to the corresponding absolute magnitude of the standard cluster for the final absolute magnitude estimation. Also, the new procedure decreased the number of columns in the final tables. We estimated the  $M_g$  absolute magnitudes for the  $(g-r)_0$  colours given in Table 4 for the cluster sample in Table 1 by combining the  $g_0$  apparent magnitudes evaluated by Eq. (2) and the true distance modulus ( $\mu_0$ ) of the cluster in question, i.e.

$$M_g = g_0 - \mu_0. \quad (3)$$

Then, we plotted the absolute magnitudes versus  $(g-r)_0$  colours. Fig. 2 shows that the absolute magnitude is colour and metallicity dependent. It increases (algebraically) with increasing metallicity and decreasing colour.

Now, we can fit the  $M_g$  absolute magnitudes to the corresponding  $[Fe/H]$  metallicity for a given  $(g-r)_0$  colour index and obtain the required calibration. This is carried out for the colour indices  $(g-r)_0 = 0.60, 0.75, 0.95, 1.05$  and  $1.20$  just for the exhibition of the procedure. The results are given in Table 5 and Fig. 3. The absolute magnitudes in the colour indices  $(g-r)_0 = 0.60, 0.75, 1.05$  and  $1.20$  could be fitted to a second degree polynomial with (squared) correlation coefficients  $R^2 \geq 0.9993$ . The range of the metallicity for the colour index  $(g-r)_0 = 0.95$  is the highest, i.e.  $-2.15 \leq [Fe/H] \leq 0.37$  dex. Hence, a third degree polynomial was necessary for the high (squared) correlation coefficient  $R^2 = 0.9994$ . The high correlation coefficients indicate accurate absolute magnitude estimation.

This procedure can be applied to any  $(g-r)_0$  colour-interval for which the sample clusters are defined. The  $(g-r)_0$  domain of the clusters are different. Hence, we adopted this interval in our study as  $0.45 \leq (g-r)_0 \leq 1.30$  where at least two clusters are defined, and we evaluated  $M_g$  absolute magnitudes for each colour. Then, we combined them with the corresponding  $[Fe/H]$  metallicities and obtained the final

calibrations. The metallicities of the clusters M13 and M3 are close to each other, i.e.  $[Fe/H]$  -1.41 and -1.50 dex, respectively. Hence, we adopted the mean of the data of these clusters in the absolute magnitude calibration. The general form of the equation for the calibrations is as follows:

$$M_g = b_0 + b_1X + b_2X^2 + b_3X^3 \quad (4)$$

where  $X = [Fe/H]$ .

$M_g$  could be fitted in terms metallicity by different degrees of polynomials. A cubic polynomial was necessary only for a limited interval, i.e.  $0.85 \leq (g-r)_0 \leq 0.96$ , for a high correlation coefficient. Whereas a quadratic or linear polynomials were sufficient for most of the colour indices, i.e.  $0.45 \leq (g-r)_0 \leq 0.84$  and  $0.97 \leq (g-r)_0 \leq 1.30$ , for a high correlation coefficient. The degree of the polynomial depends mainly on the metallicity range considered. However, despite of the large domain in metallicity,  $-2.15 \leq [Fe/H] \leq 0.37$  dex, absolute magnitudes for the colour index interval  $0.97 \leq (g-r)_0 \leq 1.15$  could be fitted by quadratic polynomials with high correlation coefficients. Then, one can say that the data presented for different clusters are homogeneous and our procedure promises accurate absolute magnitude estimation. The absolute magnitudes estimated via Eq. (3) for 86  $(g-r)_0$  colour indices and the corresponding  $b_i$  ( $i=0, 1, 2, 3$ ) coefficients are given in Table 6. However, the diagrams for the calibrations are not given in the paper for avoiding space consuming. One can use any data set taken from Table 6 depending on the desire accuracy, and apply it to stars whose iron abundances are available.

The calibration of  $M_g$  in terms of  $[Fe/H]$  is carried out for the colour interval  $0.45 \leq (g-r)_0 \leq 1.30$  mag in steps of 0.01 mag. A small step is necessary to isolate an observational error on  $g-r$  plus an error due to reddening. The origin of the mentioned errors is the trend of the red giant branch (RGB) sequence. As it is very steep, a small error in  $g-r$  implies a large change in the absolute magnitude.

Iron abundance,  $[Fe/H]$ , is not the only parameter determining the chemistry of the star but also alpha enhancement,  $[\alpha/Fe]$ , is surely important. However, as stated in Paper I, there is a correlation between two sets of abundances. Hence, we do not expect any considerable change in the numerical values of  $M_g$  in the case of addition of the alpha enhancement term in Eq. (4).

## 3.2 Application of the Method

We applied the method to five clusters with different metallicities, i.e. M15, M53, M5, NGC 5466, and NGC 7006, as explained in the following. The reason of choosing clusters

Table 5:  $M_g$  absolute magnitudes and  $[Fe/H]$  metallicities for five  $(g-r)_0$ -intervals

$(g-r)_0$ (mag)	$[Fe/H]$ (dex)	$M_g$ (mag)
0.60	-2.15	-0.083
	-1.46	0.778
	-0.78	1.669
	-0.25	2.587
0.75	-2.15	-1.156
	-1.46	-0.450
	-0.78	0.559
	-0.25	1.689
0.95	-2.15	-1.844
	-1.46	-1.246
	-0.78	-0.274
	-0.25	0.842
	0.37	3.469
1.05	-2.15	-1.951
	-1.46	-1.479
	-0.78	-0.506
	0.37	2.547
1.20	-1.46	-1.695
	-0.78	-0.629
	0.37	1.734

Table 4:  $M_g$  absolute magnitudes estimated for a set of  $(g-r)_0$  colours for six Galactic clusters used in the calibration.

Cluster →	M92	M13	M3	M71	NGC 2158	NGC 6791
$(g-r)_0$	$M_g$					
0.45	2.072	3.049	3.457	4.845	—	—
0.50	1.147	2.161	2.299	3.362	—	—
0.55	0.453	1.419	1.419	2.354	2.962	—
0.60	-0.083	0.804	0.752	1.669	2.587	—
0.65	-0.511	0.298	0.245	1.192	2.284	—
0.70	-0.862	-0.115	-0.144	0.840	1.993	—
0.75	-1.156	-0.452	-0.448	0.559	1.689	—
0.80	-1.400	-0.725	-0.692	0.316	1.373	—
0.85	-1.595	-0.947	-0.894	0.096	1.080	4.839
0.90	-1.742	-1.128	-1.066	-0.102	0.872	4.091
0.95	-1.844	-1.276	-1.215	-0.274	0.842	3.469
1.00	-1.907	-1.400	-1.344	-0.410	—	2.960
1.05	-1.951	-1.504	-1.454	-0.506	—	2.547
1.10	-2.007	-1.591	-1.546	-0.562	—	2.216
1.15	-2.127	-1.664	-1.617	-0.593	—	1.950
1.20	—	-1.722	-1.669	-0.629	—	1.734
1.25	—	-1.763	-1.703	-0.726	—	1.550
1.30	—	-1.782	-1.726	-0.963	—	1.380
1.35	—	-1.775	—	—	—	1.208

instead of individual field giants is that clusters provide absolute magnitudes for comparison with the ones estimated by means of our method. The distance modulus, colour excess and metallicity of the clusters are given in Table 7, whereas the  $g$  magnitudes and  $g-r$  colours are presented in Table 8 and they are calibrated in Fig. 4. The  $g$  and  $g-r$  data of the clusters are taken from An et al. (2008). Also the colour excesses and the distance moduli of all clusters and the metallicities of M15 and M5 are taken from An et al. (2008). Whereas the metallicities of three clusters, M53, NGC 5466, and NGC 7006, are taken from the authors cited in Table 7. An et al. (2008) claimed  $[Fe/H] = -1.99$  and  $-1.48$  dex for the clusters M53 and NGC 7006, respectively. Whereas the metallicities in Santos & Piatti (2004), i.e.  $[Fe/H] = -1.88$  and  $-1.35$  dex, provide more accurate absolute magnitudes. The metallicities cited by An et al. (2008) and Rosenberg et al. (1999) for the cluster NGC 5466 are  $[Fe/H] = -2.22$  and  $-2.13 \pm 0.36$  dex, respectively. Here again, the metallicity in Rosenberg et al. (1999) plus its error, i.e.  $[Fe/H] = -1.17$  dex, provides more accurate absolute magnitudes.

We evaluated the  $M_g$  absolute magnitude by means of the Eq. (4) for a set of  $(g-r)_0$  colour indices where the clusters are defined. The results are presented in Table 9. The columns give: (1)  $(g-r)_0$  colour index, (2)  $(M_g)_{cl}$ , absolute

Table 7: Data for the clusters used for the application of the method.

Cluster	$E(B-V)$	$(g-M_g)_0$	$[Fe/H]$	Ref.
M15	0.10	15.25	-2.42	1
M53	0.02	16.25	-1.88	2
M5	0.03	14.42	-1.26	1
NGC5466	0.00	16.00	-1.77	3
NGC7006	0.05	18.09	-1.35	2

(1) An et al. (2008), (2) Santos & Piatti (2004), (3) Rosenberg et al. (1999).

magnitude for a cluster estimated by its colour magnitude diagram, (3)  $(M_g)_{ev}$ , the absolute magnitude estimated by the procedure (4)  $\Delta M$ , absolute magnitude residuals. Also, the metallicity for each cluster is indicated near the name of the cluster. The differences between the absolute magnitudes estimated by the procedure presented in this study and the ones evaluated via the colour magnitudes of the clusters (the residuals) lie between  $-0.28$  and  $+0.43$  mag. However, the range of 94% of the absolute magnitude residuals is shorter, i.e.  $0.1 < M_g \leq 0.4$  mag. The mean and the standard deviation of the residuals are  $<\Delta M> = 0.169$  and  $\sigma = 0.140$  mag, respectively. The distribution of the residuals are given in Table 10 and Fig. 4.

Table 6:  $M_g$  absolute magnitudes estimated for six Galactic clusters and the numerical values of  $b_i$  ( $i = 0, 1, 2, 3$ ) coefficients in Eq. (3). The absolute magnitudes and metallicities of the clusters M13 and M3 were combined in the evaluation of  $b_i$  coefficients. The last column gives the range of the metallicity  $[Fe/H]$  (dex) for the star whose absolute magnitude would be estimated.  $R^2$  is the square of the correlation coefficient.

Cluster →	M92	M13+M3	M71	NGC 2158	NGC 6791							
$(g-r)_0$			$M_g$			$b_0$	$b_1$	$b_2$	$b_3$	$R^2$	$[Fe/H]$ -interval	
0.45	2.072	3.253	4.845	—	—	7.2311	3.4339	0.4810	—	1	[-2.15, -0.78]	
0.46	1.864	3.029	4.502	—	—	6.6217	3.0054	0.3686	—	1	[-2.15, -0.78]	
0.47	1.669	2.815	4.183	—	—	6.0756	2.6404	0.2747	—	1	[-2.15, -0.78]	
0.48	1.484	2.611	3.888	—	—	5.2247	1.7538	—	—	0.9980	[-2.15, -0.78]	
0.49	1.311	2.416	3.615	—	—	4.9045	1.6812	—	—	0.9990	[-2.15, -0.78]	
0.50	1.147	2.230	3.362	—	—	4.6084	1.6161	—	—	0.9996	[-2.15, -0.78]	
0.51	0.992	2.052	3.127	—	—	4.3349	1.5582	—	—	0.9998	[-2.15, -0.78]	
0.52	0.846	1.883	2.911	—	—	4.0823	1.5068	—	—	1	[-2.15, -0.78]	
0.53	0.708	1.721	2.710	—	—	3.8493	1.4615	—	—	1	[-2.15, -0.78]	
0.54	0.577	1.566	2.525	—	—	3.6344	1.4219	—	—	1	[-2.15, -0.78]	
0.55	0.453	1.419	2.354	2.962	—	3.2590	1.1272	-0.0842	—	0.9997	[-2.15, -0.25]	
0.56	0.335	1.278	2.195	2.877	—	3.2028	1.2831	-0.0241	—	1	[-2.15, -0.25]	
0.57	0.223	1.144	2.048	2.799	—	3.1513	1.4305	0.0322	—	1	[-2.15, -0.25]	
0.58	0.116	1.016	1.912	2.724	—	3.1036	1.5689	0.0846	—	0.9998	[-2.15, -0.25]	
0.59	0.014	0.894	1.786	2.654	—	3.0586	1.6982	0.1330	—	0.9996	[-2.15, -0.25]	
0.60	-0.083	0.778	1.669	2.587	—	3.0158	1.8181	0.1774	—	0.9993	[-2.15, -0.25]	
0.61	-0.176	0.667	1.560	2.523	—	2.9743	1.9285	0.2180	—	0.9991	[-2.15, -0.25]	
0.62	-0.265	0.561	1.458	2.461	—	2.9335	2.0292	0.2547	—	0.9989	[-2.15, -0.25]	
0.63	-0.350	0.460	1.364	2.401	—	2.8927	2.1202	0.2875	—	0.9988	[-2.15, -0.25]	
0.64	-0.432	0.364	1.275	2.342	—	2.8515	2.2015	0.3167	—	0.9987	[-2.15, -0.25]	
0.65	-0.511	0.272	1.192	2.284	—	2.8093	2.2731	0.3421	—	0.9987	[-2.15, -0.25]	
0.66	-0.586	0.184	1.114	2.226	—	2.7658	2.3352	0.3640	—	0.9988	[-2.15, -0.25]	
0.67	-0.659	0.100	1.040	2.168	—	2.7206	2.3878	0.3824	—	0.9988	[-2.15, -0.25]	
0.68	-0.729	0.020	0.970	2.110	—	2.6735	2.4313	0.3975	—	0.9989	[-2.15, -0.25]	
0.69	-0.797	-0.056	0.904	2.052	—	2.6242	2.4658	0.4094	—	0.9991	[-2.15, -0.25]	
0.70	-0.862	-0.130	0.840	1.993	—	2.5726	2.4916	0.4183	—	0.9992	[-2.15, -0.25]	
0.71	-0.925	-0.199	0.780	1.934	—	2.5187	2.5093	0.4244	—	0.9993	[-2.15, -0.25]	
0.72	-0.986	-0.266	0.722	1.874	—	2.4623	2.5191	0.4278	—	0.9994	[-2.15, -0.25]	
0.73	-1.045	-0.330	0.666	1.813	—	2.4037	2.5216	0.4287	—	0.9996	[-2.15, -0.25]	
0.74	-1.101	-0.391	0.612	1.751	—	2.3428	2.5174	0.4274	—	0.9997	[-2.15, -0.25]	
0.75	-1.156	-0.450	0.559	1.689	—	2.2799	2.5071	0.4242	—	0.9998	[-2.15, -0.25]	
0.76	-1.209	-0.506	0.508	1.626	—	2.2152	2.4914	0.4192	—	0.9998	[-2.15, -0.25]	
0.77	-1.259	-0.560	0.458	1.563	—	2.1491	2.4709	0.4128	—	0.9999	[-2.15, -0.25]	
0.78	-1.308	-0.611	0.410	1.500	—	2.0819	2.4466	0.4052	—	1	[-2.15, -0.25]	
0.79	-1.355	-0.661	0.363	1.436	—	2.0140	2.4193	0.3968	—	1	[-2.15, -0.25]	
0.80	-1.400	-0.708	0.316	1.373	—	1.9460	2.3900	0.3880	—	1	[-2.15, -0.25]	
0.81	-1.442	-0.754	0.270	1.311	—	1.8784	2.3595	0.3789	—	1	[-2.15, -0.25]	
0.82	-1.484	-0.798	0.226	1.251	—	1.8119	2.3291	0.3701	—	1	[-2.15, -0.25]	
0.83	-1.523	-0.840	0.182	1.191	—	1.7471	2.2999	0.3619	—	1	[-2.15, -0.25]	
0.84	-1.560	-0.881	0.139	1.134	—	1.6848	2.2731	0.3547	—	0.9999	[-2.15, -0.25]	
0.85	-1.595	-0.920	0.096	1.080	4.839	2.3718	5.2610	3.2390	0.7703	0.9963	[-2.15, 0.37]	
0.86	-1.628	-0.958	0.055	1.029	4.679	2.2861	5.1147	3.1103	0.7367	0.9964	[-2.15, 0.37]	
0.87	-1.660	-0.995	0.014	0.982	4.524	2.2046	4.9722	2.9820	0.7027	0.9965	[-2.15, 0.37]	
0.88	-1.689	-1.030	-0.026	0.939	4.374	2.1278	4.8327	2.8525	0.6678	0.9967	[-2.15, 0.37]	
0.89	-1.717	-1.064	-0.064	0.902	4.230	2.0561	4.6956	2.7203	0.6314	0.9969	[-2.15, 0.37]	
0.90	-1.742	-1.097	-0.102	0.872	4.091	1.9900	4.5600	2.5836	0.5929	0.9972	[-2.15, 0.37]	
0.91	-1.766	-1.129	-0.139	0.848	3.957	1.9300	4.4252	2.4407	0.5518	0.9976	[-2.15, 0.37]	
0.92	-1.788	-1.159	-0.174	0.832	3.828	1.8767	4.2902	2.2897	0.5074	0.9980	[-2.15, 0.37]	
0.93	-1.808	-1.189	-0.209	0.825	3.704	1.8307	4.1541	2.1285	0.4590	0.9984	[-2.15, 0.37]	
0.94	-1.827	-1.218	-0.242	0.828	3.584	1.7925	4.0161	1.9552	0.4060	0.9989	[-2.15, 0.37]	
0.95	-1.844	-1.246	-0.274	0.842	3.469	1.7628	3.8750	1.7677	0.3477	0.9994	[-2.15, 0.37]	
0.96	-1.859	-1.272	-0.304	0.868	3.359	1.7422	3.7300	1.5637	0.2833	0.9998	[-2.15, 0.37]	
0.97	-1.873	-1.299	-0.333	—	3.253	1.8965	3.3483	0.7489	—	0.9991	[-2.15, 0.37]	
0.98	-1.885	-1.324	-0.360	—	3.151	1.8225	3.2835	0.7315	—	0.9992	[-2.15, 0.37]	
0.99	-1.897	-1.348	-0.386	—	3.053	1.7515	3.2213	0.7148	—	0.9993	[-2.15, 0.37]	
1.00	-1.907	-1.372	-0.410	—	2.960	1.6835	3.1616	0.6988	—	0.9995	[-2.15, 0.37]	
1.01	-1.917	-1.395	-0.432	—	2.870	1.6185	3.1042	0.6834	—	0.9996	[-2.15, 0.37]	
1.02	-1.925	-1.417	-0.453	—	2.784	1.5564	3.0488	0.6685	—	0.9997	[-2.15, 0.37]	
1.03	-1.934	-1.438	-0.472	—	2.701	1.4972	2.9954	0.6539	—	0.9998	[-2.15, 0.37]	
1.04	-1.942	-1.459	-0.490	—	2.622	1.4409	2.9438	0.6396	—	0.9999	[-2.15, 0.37]	
1.05	-1.951	-1.479	-0.506	—	2.547	1.3873	2.8937	0.6253	—	0.9999	[-2.15, 0.37]	
1.06	-1.960	-1.498	-0.520	—	2.475	1.3366	2.8449	0.6110	—	1	[-2.15, 0.37]	
1.07	-1.969	-1.517	-0.532	—	2.406	1.2885	2.7974	0.5965	—	1	[-2.15, 0.37]	
1.08	-1.980	-1.535	-0.543	—	2.339	1.2431	2.7508	0.5816	—	1	[-2.15, 0.37]	
1.09	-1.993	-1.552	-0.553	—	2.276	1.2008	2.7051	0.5663	—	1	[-2.15, 0.37]	
1.10	-2.007	-1.568	-0.562	—	2.216	1.1599	2.6601	0.5503	—	1	[-2.15, 0.37]	
1.11	-2.024	-1.584	-0.569	—	2.158	1.1221	2.6155	0.5335	—	0.9998	[-2.15, 0.37]	
1.12	-2.044	-1.599	-0.576	—	2.103	1.0865	2.5713	0.5159	—	0.9998	[-2.15, 0.37]	
1.13	-2.068	-1.614	-0.582	—	2.050	1.0533	2.5273	0.4971	—	0.9997	[-2.15, 0.37]	
1.14	-2.095	-1.627	-0.587	—	1.999	1.0223	2.4834	0.4771	—	0.9996	[-2.15, 0.37]	
1.15	-2.127	-1.640	-0.593	—	1.950	0.9933	2.4394	0.4557	—	0.9996	[-2.15, 0.37]	
1.16	—	-1.653	-0.598	—	1.904	1.0018	2.3131	0.3359	—	1	[-1.46, 0.37]	
1.17	—	-1.664	-0.604	—	1.859	0.9761	2.2699	0.3128	—	1	[-1.46, 0.37]	
1.18	—	-1.675	-0.611	—	1.816	0.9505	2.2300	0.2923	—	1	[-1.46, 0.37]	
1.19	—	-1.686	-0.619	—	1.774	0.9247	2.1938	0.2747	—	1	[-1.46, 0.37]	
1.20	—	-1.695	-0.629	—	1.734	0.8983	2.1619	0.2608	—	1	[-1.46, 0.37]	
1.21	—	-1.704	-0.642	—	1.695	0.8708	2.1348	0.2509	—	1	[-1.46, 0.37]	
1.22	—	-1.712	-0.657	—	1.657	0.8418	2.1131	0.2458	—	1	[-1.46, 0.37]	
1.23	—	-1.720	-0.675	—	1.621	0.8108	2.0975	0.2462	—	1	[-1.46, 0.37]	
1.24	—	-1.727	-0.698	—	1.585	0.7773	2.0888	0.2528	—	1	[-1.46, 0.37]	
1.25	—	-1.733	-0.726	—	1.550	0.7407	2.0878	0.2665	—	1	[-1.46, 0.37]	
1.26	—	-1.738	-0.759	—	1.515	0.7005	2.0953	0.2881	—	1	[-1.46, 0.37]	
1.27	—	-1.743	-0.798	—	1.481	0.6559	2.1124	0.3186	—	1	[-1.46, 0.37]	
1.28	—	-1.747	-0.844	—	1.447	0.6064	2.1401	0.3590	—	1	[-1.46, 0.37]	
1.29	—	-1.751	-0.899	—	1.414	0.5512	2.1793	0.4103	—	1	[-1.46, 0.37]	
1.30	—	-1.754	-0.963	—	1.380	0.4897	2.2315	0.4738	—	1	[-1.46, 0.37]	

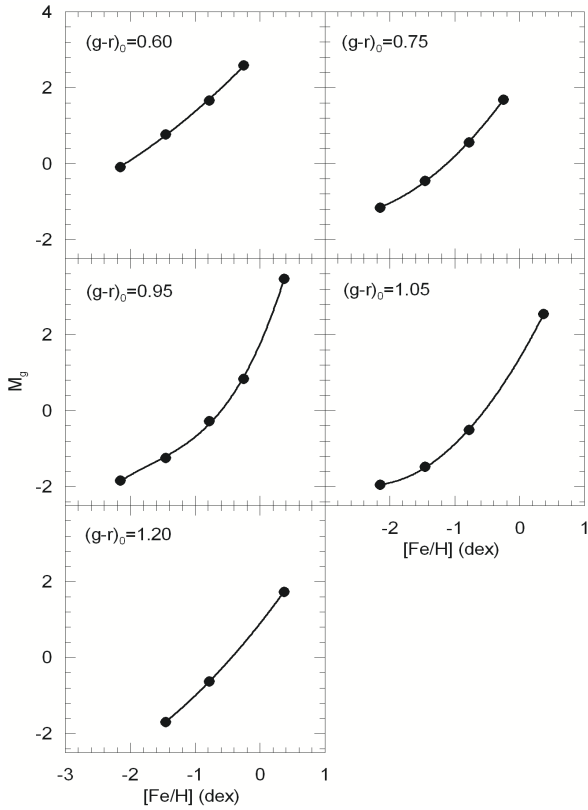


Figure 3: Calibration of the absolute magnitude  $M_g$  as a function of metallicity  $[Fe/H]$  for five colour-indices.

The absolute magnitudes on the RGB at a given colour and metallicity do not change linearly or quadratically with age. Instead, the absolute magnitudes gets rapidly fainter for young (and massive) stars with a certain  $g-r$  and  $[Fe/H]$ , but shows virtually the same absolute magnitude for all old stars, i.e.  $t > 6$  Gyrs. That is, the gradient of the absolute magnitude respect to a given colour and metallicity is greater for a young star relative to an old one.

## 4 Summary and Discussion

We presented an absolute magnitude calibration for giants based on the colour-magnitude diagrams of six Galactic clusters with different metallicities, i.e. M92, M13, M3, M71, NGC 6791 and NGC 2158. All the clusters were observed in the  $u'g'r'i'z'$  passbands by Clem, Vanden Berg & Stetson (2008) except the cluster NGC 2158 which is observed in the  $ugriz$  passbands by Smolinski et al. (2011). We used the transformations of Rider et al. (2004) and transformed the  $g'$  and  $g'-r'$  data in Clem, Vanden Berg & Stetson (2008) to the  $g$  and  $g-r$  data. Thus, we obtained a homogeneous set of

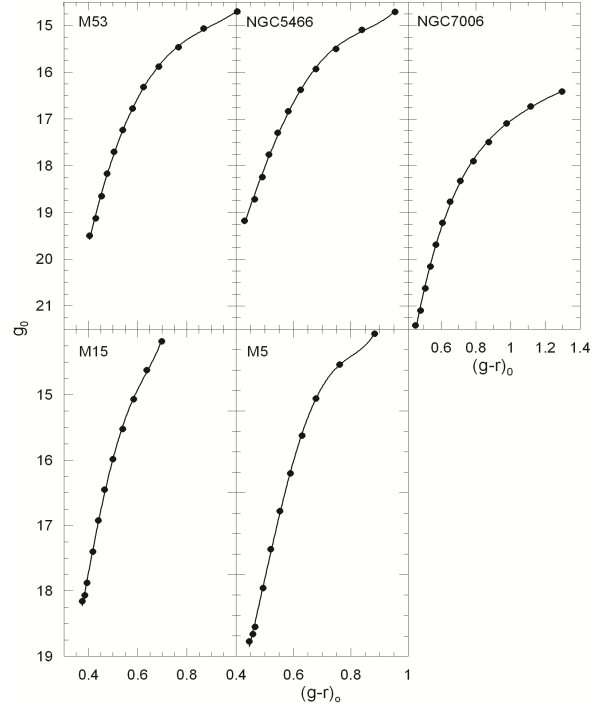


Figure 4:  $g_0, (g-r)_0$  colour-apparent magnitude diagrams for the Galactic clusters used for the application of the procedure.

data in SDSS system for absolute magnitude calibration. We combined the calibrations between  $g_0$  and  $(g-r)_0$  for each cluster with their true distance modulus and evaluated a set of absolute magnitudes for the  $(g-r)_0$  range of each clusters. Then, we fitted the  $M_g$  absolute magnitudes in terms of iron metallicity,  $[Fe/H]$ , by different degrees of polynomials for a given  $(g-r)_0$  colour index. Our absolute magnitude calibrations cover the range  $0.45 \leq (g-r)_0 \leq 1.30$ . However, not all the clusters could be considered for each  $(g-r)_0$  colour index in this interval due to different  $(g-r)_0$  domains of the clusters. The limited interval that all the clusters were considered is  $0.85 \leq (g-r)_0 \leq 0.96$ . Also, this interval is the unique interval where the highest degree ( $n=3$ ) of polynomial was fitted. A linear or quadratic polynomial was sufficient for the colour intervals  $0.45 \leq (g-r)_0 \leq 0.84$  and  $0.97 \leq (g-r)_0 \leq 1.30$  for a high correlation coefficient.

We applied the procedure to another set of Galactic cluster, i.e. M15, M53, M5, NGC 5466 and NGC 7006. The reason for this chose is that a cluster provides absolute magnitude for comparison with the ones estimated by means of our procedure. We used the equations of Fan (1999) for dereddening of the colour and magnitudes, and the calibration in Eq. (4) for evaluation a set of  $M_g$  absolute magnitudes for each cluster in their  $(g-r)_0$  domain.



Table 8: Fiducial giant sequences for the Galactic clusters used in the application of the procedure.

$g-r$	$g$	$(g-r)_0$	$g_0$	$g-r$	$g$	$(g-r)_0$	$g_0$
M15				M5 (cont.)			
0.803	14.553	0.697	14.181	0.552	16.802	0.520	16.690
0.743	14.993	0.637	14.621	0.525	17.275	0.493	17.163
0.689	15.439	0.583	15.067	0.498	17.748	0.466	17.636
0.645	15.895	0.539	15.523	0.490	17.840	0.458	17.728
0.604	16.354	0.498	15.982	0.477	17.927	0.445	17.815
0.571	16.821	0.465	16.449	NGC5466			
0.546	17.296	0.440	16.924	0.954	14.704	0.954	14.704
0.523	17.773	0.417	17.401	0.839	15.089	0.839	15.089
0.500	18.250	0.394	17.878	0.748	15.498	0.748	15.498
0.490	18.440	0.384	18.068	0.679	15.929	0.679	15.929
0.480	18.530	0.374	18.158	0.625	16.375	0.625	16.375
M53				0.582	16.832	0.582	16.832
1.026	14.776	1.005	14.702	0.545	17.295	0.545	17.295
0.890	15.140	0.869	15.066	0.514	17.764	0.514	17.764
0.788	15.538	0.767	15.464	0.491	18.241	0.491	18.241
0.708	15.958	0.687	15.884	0.465	18.715	0.465	18.715
0.645	16.395	0.624	16.321	0.429	19.179	0.429	19.179
0.600	16.850	0.579	16.776	NGC7006			
0.560	17.310	0.539	17.236	1.348	16.598	1.295	16.412
0.525	17.775	0.504	17.701	1.168	16.918	1.115	16.732
0.497	18.247	0.476	18.173	1.030	17.280	0.977	17.094
0.475	18.725	0.454	18.651	0.927	17.677	0.874	17.491
0.451	19.201	0.430	19.127	0.839	18.089	0.786	17.903
0.425	19.575	0.404	19.501	0.763	18.513	0.710	18.327
M5				0.703	18.953	0.650	18.767
0.915	14.165	0.883	14.053	0.659	19.409	0.606	19.223
0.792	14.542	0.760	14.430	0.622	19.872	0.569	19.686
0.710	14.960	0.678	14.848	0.591	20.341	0.538	20.155
0.661	15.411	0.629	15.299	0.562	20.812	0.509	20.626
0.621	15.871	0.589	15.759	0.534	21.284	0.481	21.098
0.584	16.334	0.552	16.222	0.504	21.604	0.451	21.418

Table 9: Absolute magnitudes ( $(M_g)_{ev}$ ) and residuals ( $\Delta M$ ) estimated by the procedure explained in our work.  $(M_g)_{cl}$  denotes the absolute magnitude evaluated by means of the colour-magnitude diagram of the cluster.

(1)	(2)	(3)	(4)	(1)	(2)	(3)	(4)	(1)	(2)	(3)	(4)
$(g-r)_0$	$(M_g)_{cl}$	$(M_g)_{ev}$	$\Delta M$	$(g-r)_0$	$(M_g)_{cl}$	$(M_g)_{ev}$	$\Delta M$	$(g-r)_0$	$(M_g)_{cl}$	$(M_g)_{ev}$	$\Delta M$
M15 ([Fe/H]=-2.42 dex)				M5 (cont.)				NGC7006 ([Fe/H]=-1.35 dex)			
0.45	1.517	1.738	-0.221	0.50	2.627	2.572	0.054	0.45	3.470	3.472	-0.002
0.47	1.176	1.295	-0.118	0.52	2.315	2.184	0.132	0.47	3.102	3.012	0.090
0.50	0.719	0.697	0.021	0.55	1.863	1.705	0.158	0.50	2.596	2.427	0.169
0.52	0.452	0.436	0.016	0.57	1.578	1.400	0.178	0.52	2.287	2.048	0.239
0.55	0.109	0.038	0.071	0.60	1.188	1.007	0.181	0.55	1.864	1.584	0.280
0.57	-0.085	-0.122	0.037	0.62	0.955	0.781	0.174	0.57	1.606	1.279	0.327
0.60	-0.334	-0.345	0.011	0.65	0.654	0.488	0.165	0.60	1.253	0.885	0.369
0.62	-0.481	-0.486	0.005	0.67	0.485	0.319	0.166	0.62	1.039	0.658	0.381
0.65	-0.692	-0.688	-0.004	0.70	0.279	0.097	0.182	0.65	0.747	0.364	0.383
0.67	-0.842	-0.818	-0.023	0.72	0.171	-0.033	0.204	0.67	0.570	0.194	0.376
0.70	-1.109	-1.007	-0.102	0.75	0.047	-0.206	0.252	0.70	0.329	-0.029	0.358
M53 ([Fe/H]=-1.88 dex)				0.77	-0.017	-0.309	0.292	0.72	0.183	-0.159	0.342
0.45	2.400	2.475	-0.075	0.80	-0.094	-0.449	0.355	0.75	-0.016	-0.332	0.316
0.47	2.043	2.083	-0.039	0.82	-0.143	-0.535	0.392	0.77	-0.136	-0.434	0.298
0.50	1.557	1.570	-0.014	0.85	-0.230	-0.656	0.426	0.80	-0.301	-0.573	0.273
0.52	1.263	1.250	0.013	0.87	-0.308	-0.732	0.423	0.82	-0.401	-0.658	0.257
0.55	0.866	0.842	0.023	NGC5466 ([Fe/H]=-1.77 dex)				0.85	-0.538	-0.723	0.185
0.57	0.628	0.576	0.052	0.45	2.872	2.660	0.212	0.87	-0.621	-0.802	0.181
0.60	0.309	0.225	0.085	0.47	2.526	2.263	0.263	0.90	-0.737	-0.916	0.179
0.62	0.120	0.019	0.102	0.50	2.031	1.748	0.283	0.92	-0.808	-0.990	0.183
0.65	-0.131	-0.255	0.124	0.52	1.720	1.415	0.305	0.95	-0.907	-1.102	0.195
0.67	-0.279	-0.417	0.138	0.55	1.284	1.000	0.284	0.97	-0.968	-1.259	0.290
0.70	-0.474	-0.633	0.160	0.57	1.016	0.720	0.296	1.00	-1.055	-1.311	0.256
0.72	-0.587	-0.762	0.174	0.60	0.649	0.354	0.296	1.02	-1.109	-1.341	0.232
0.75	-0.737	-0.934	0.197	0.62	0.429	0.140	0.290	1.05	-1.187	-1.380	0.193
0.77	-0.825	-1.037	0.212	0.65	0.136	-0.142	0.279	1.07	-1.236	-1.401	0.165
0.80	-0.941	-1.176	0.235	0.67	-0.035	-0.308	0.273	1.10	-1.306	-1.428	0.122
0.82	-1.010	-1.259	0.248	0.70	-0.257	-0.527	0.270	1.12	-1.351	-1.445	0.093
0.85	-1.104	-1.189	0.085	0.72	-0.384	-0.656	0.272	1.15	-1.416	-1.469	0.053
0.87	-1.162	-1.273	0.111	0.75	-0.545	-0.829	0.284	1.17	-1.458	-1.518	0.060
0.90	-1.245	-1.391	0.146	0.77	-0.635	-0.931	0.296	1.20	-1.518	-1.545	0.027
0.92	-1.299	-1.468	0.168	0.80	-0.750	-1.069	0.318	1.22	-1.556	-1.563	0.007
0.95	-1.383	-1.585	0.202	0.82	-0.817	-1.151	0.334	1.25	-1.611	-1.592	-0.019
0.97	-1.441	-1.751	0.310	0.85	-0.909	-1.064	0.155	1.27	-1.645	-1.615	-0.030
1.00	-1.537	-1.790	0.253	0.87	-0.969	-1.151	0.181	1.30	-1.693	-1.659	-0.033
M5 ([Fe/H]=-1.26 dex)				0.90	-1.067	-1.275	0.208	—	—	—	—
0.45	3.392	3.668	-0.276	0.92	-1.143	-1.357	0.214	—	—	—	—
0.47	3.093	3.185	-0.092	0.95	-1.282	-1.486	0.204	—	—	—	—

Table 11: Absolute magnitudes estimated by altering the metallicity as  $[Fe/H] + \Delta[Fe/H]$ . The numerical values of  $[Fe/H]$  are indicated in the last column. The absolute magnitudes in column (1) are the original ones taken from Table 9, whereas those in the columns (2) – (5) correspond to the increments 0.05, 0.10, 0.15, and 0.20 dex. The differences between the original absolute magnitudes and those evaluated by means of the metallicity increments are given in columns (6)–(9).

$M_g$						$\Delta M$				
$(g-r)_0$	(1)	(2)	(3)	(4)	(5)	(6)	(7)	(8)	(9)	
0.50	1.570	1.651	1.732	1.813	1.893	0.081	0.162	0.242	0.323	$[Fe/H] = -1.88 + \Delta[Fe/H]$
0.65	-0.255	-0.205	-0.153	-0.099	-0.044	0.050	0.102	0.156	0.211	
0.85	-1.189	-1.130	-1.075	-1.024	-0.977	0.060	0.115	0.165	0.212	
0.50	2.572	2.653	2.734	2.815	2.895	0.081	0.162	0.242	0.323	$[Fe/H] = -1.26 + \Delta[Fe/H]$
0.65	0.488	0.560	0.633	0.708	0.784	0.071	0.145	0.219	0.296	
0.85	-0.656	-0.616	-0.575	-0.531	-0.483	0.039	0.081	0.125	0.173	
0.50	1.748	1.829	1.910	1.990	2.071	0.081	0.162	0.242	0.323	$[Fe/H] = -1.77 + \Delta[Fe/H]$
0.65	-0.142	-0.088	-0.033	0.025	0.084	0.054	0.110	0.167	0.226	
0.85	-1.064	-1.014	-0.968	-0.926	-0.885	0.050	0.096	0.139	0.179	
0.50	2.427	2.507	2.588	2.669	2.750	0.081	0.162	0.242	0.323	$[Fe/H] = -1.35 + \Delta[Fe/H]$
0.65	0.364	0.432	0.502	0.574	0.648	0.068	0.138	0.210	0.284	
0.85	-0.723	-0.686	-0.648	-0.608	-0.566	0.037	0.075	0.114	0.156	
1.00	-1.311	-1.246	-1.177	-1.104	-1.028	0.065	0.134	0.207	0.283	
1.20	-1.545	-1.471	-1.397	-1.320	-1.243	0.074	0.148	0.225	0.302	

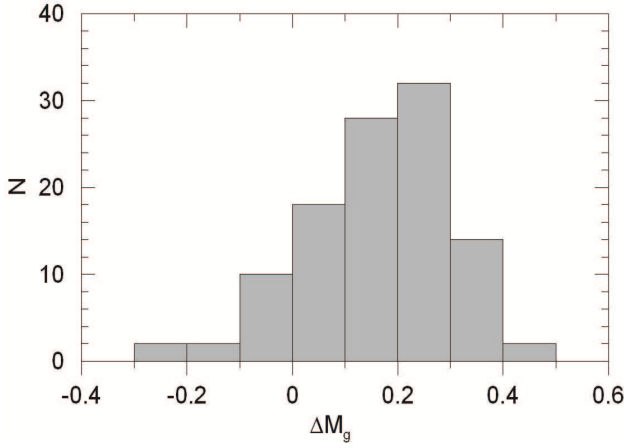


Figure 5: Histogram of the residuals.

We compared the absolute magnitudes estimated by this procedure with those evaluated via combination of the fiducial  $g_0$ ,  $(g-r)_0$  sequence and the true distance modulus for each cluster. The residuals lie between -0.28 and +0.43 mag. However, the range of 94% of them is smaller, i.e.  $0.1 < M_g \leq 0.4$  mag. The mean and the standard deviation of all the residuals are  $\langle \Delta M \rangle = 0.169$  and  $\sigma = 0.140$  mag, respectively. The range of the residuals in Paper I was greater than the one in this study, i.e.  $-0.61 < \Delta M_V < +0.66$  mag. Also, the mean and the standard deviation of the residuals in a smaller range,  $-0.4 \leq \Delta M \leq +0.4$ , which consists of 91% of the residuals were  $\langle \Delta M \rangle = 0.05$  and  $\sigma = 0.19$  mag, respectively. Comparison of the statistical results presented in two studies shows that there is a small improvement on the results in this study with respect to the former one. As claimed in Paper I, there was an improvement on the results therein respect to the ones of Hog & Flynn (1998). Hence, the same improve-

Table 10: Distribution of the residuals.  $N$  denotes the number of stars.

$\Delta M$ -interval	$\langle \Delta M \rangle$	$N$
$(-0.3, -0.2]$	-0.249	2
$(-0.2, -0.1]$	-0.110	2
$(-0.1, 0.0]$	-0.033	10
$(0.0, 0.1]$	0.045	18
$(0.1, 0.2]$	0.163	28
$(0.2, 0.3]$	0.258	32
$(0.3, 0.4]$	0.348	14
$(0.4, 0.5]$	0.425	2

ment holds for this study. The same improvement holds also for the work of Ljunggren & Oja (1966).

Although age plays an important role in the trend of the fiducial sequence of the RGB, we have not used it as a parameter in the calibration of absolute magnitude. Another problem may originate from the red clump (RC) stars. These stars lie very close to the RGB but they present a completely different group of stars. Table 10 and Fig. 5 summarize how reliable are our absolute magnitudes. If age and possibly the mix with RC stars would affect our results this should show up, i.e. the range of the residuals would be greater and their distributions would be multimodal. Whereas, in our study their range is small and the histogram of the residuals in Fig. 5 is almost symmetric resembling a Gaussian distribution. Additionally, we should add that the fiducial sequences used in our study were properly selected as RGB. However, the researchers should identify and exclude the RC stars when they apply our calibrations to the field stars.

The accuracy of the estimated absolute magnitudes depends mainly on the accuracy of the metallicity. We altered

the metallicity by  $[Fe/H] + \Delta[Fe/H]$  in the evaluation of the absolute magnitudes by the procedure presented in our study and checked its effect on the absolute magnitude. We adopted  $[Fe/H] = -1.88, -1.26, -1.77, -1.35$  dex and  $\Delta[Fe/H] = 0.05, 0.10, 0.15, 0.20$  dex and re-evaluated the absolute magnitudes for 14  $(g - r)_0$  colour indices for this purpose. The differences between the absolute magnitudes evaluated in this way and the corresponding ones evaluated without  $\Delta[Fe/H]$  increments are given in Table 11. The maximum difference in absolute magnitude is  $\sim 0.3$  mag corresponding to the metallicity increment  $\Delta[Fe/H] = 0.20$  dex. The mean error in metallicity for 42 globular and 33 open clusters in the catalogue of Santos & Piatti (2004) is  $\sigma = 0.19$  dex. If we assume the same error for the field stars, the probable error in  $M_g$  magnitudes would be less than 0.3 mag.

The absolute magnitude could be calibrated as a function of ultraviolet excess, instead of metallicity. However, the ultraviolet magnitudes can not be provided easily. Whereas, metallicity can be derived by different methods, such as by means of atmospheric model parameters of a star, a procedure which is applied rather extensively in large surveys such as RAVE. In such cases, one needs to transform the calibration from SDSS to the system in question. The age is a secondary parameter for the old clusters and does not influence much the position of their RGB. The youngest cluster in our paper is NGC 2158 with age 2 Gyr (Carraro, Girardi & Marigo 2002). However, the field stars may be much younger. We should remind that the derived relations are applicable to stars older than 2 Gyr. For clarification of this argument, let a star younger than 2 Gyr be with colour  $0.55 < (g - r)_0 < 0.86$  mag. This star will be more metal-rich than the stars in the cluster NGC 2158, and according to the positions of the colour-absolute magnitude diagrams of the clusters in Fig. 2, it will be absolutely fainter than a star in the cluster NGC 2158 of the same colour. Then, one needs to *extrapolate* the corresponding absolute magnitude-metallicity diagram in Fig. 3 for its absolute magnitude evaluation (one of the two panels at the top depending on its colour). However, extrapolation may result in erroneous absolute  $M_g$  magnitudes.

We conclude that the  $M_g$  absolute magnitudes of the red giants can be estimated with an accuracy of  $\Delta M \leq 0.3$  mag, provided that their  $[Fe/H]$  metallicities are known.

## Acknowledgments

We thank to the anonymous reviewer for his/her comments. This research has made use of NASA's Astrophysics Data System and the SIMBAD database, operated at CDS, Strasbourg, France

## References

- An, D. et al. 2008, ApJS, 179, 326
- Bilir, S., Karaali, S., Ak, S., Yaz, E., Cabrera-Lavers, A., Coşkunoğlu, K. B. 2008, MNRAS, 390, 1569
- Bilir, S., Karaali, S., Ak, S., Coşkunoğlu, K. B., Yaz, E., Cabrera-Lavers, A. 2009, MNRAS, 396, 1589
- Breddels, M. A., et al. 2010, A&A, 511A, 90
- Carraro, G., Girardi, L., Marigo, P. 2002, MNRAS, 332, 705
- Chen, B., et al. 2001 ApJ, 553, 184
- Clem, J. L., Vanden Berg, D. A., Stetson, P. B. 2008, AJ, 135, 682
- ESA, 1997, The Hipparcos and Tycho Catalogues, ESA SP-1200.ESA Noordwijk
- Fan, X. 1999, AJ, 117, 2528
- Fukugita, M., Ichikawa, T., Gunn, J. E., Doi, M., Shimasaku, K., Schneider, D. P. 1996, AJ, 111, 1748
- Gratton, R. G., Fusi Pecci, F., Carretta, E., Clementini, G., Corsi, C. E., Lattanzi, M. 1997, ApJ, 491, 749
- Harris, W. E. 1996, AJ, 112, 1487
- Harris, W. E. 2010, arXiv:1012.3224H
- Hodder, P. J. C., Nemec, J. M., Richer, H. B., Fahlman, G. G. 1992, AJ, 103, 460
- Hog, E., Flynn, C. 1998, MNRAS, 294, 28
- Karaali, S., Karataş, Y., Bilir, S., Ak, S. G., Hamzaoglu, E. 2003, PASA, 20, 270
- Karaali, S., Bilir, S., Yaz Gökçe, E., 2012, PASA (Paper I, accepted for publication, arXiv:1204.4291K)
- Laird, J. B., Carney, B. W., Latham, D. W. 1988, AJ, 96, 1908
- Ljunggren, B., Oja, T. 1966, IAUS, 24, 317
- Marigo, P., Girardi, L., Bressan, A., Groenewegen, M. A. T., Silva, L., Granato, G. L. 2008, A&A, 482, 883
- Nissen, P. E., Schuster, W. J. 1991, A&A, 251, 457
- Phleps, S., Meisenheimer, K., Fuchs, B., Wolf, C. 2000, A&A, 356, 108
- Rider, C. J., Tucker, D. L., Smith, J. A., Stoughton, C., Al-lam, S. S., Neilsen, Eric, H. Jr. 2004, AJ, 127, 2210

- Rosenberg, A., Saviane, I., Piotto, G., Aparicio, A. 1999, AJ, 118, 2306
- Sandage, A., Lubin, L. M., VandenBerg, D. A. 2003, PASP, 115, 1187
- Santos, J. F. C. Jr., Piatti, A. E. 2004, A&A, 428, 79
- Siegel, M. H., Majewski, S. R., Reid, I. N., Thompson, I. B. 2002, ApJ, 578, 151
- Smolinski, J. P., et al. 2011, AJ, 141, 89
- VandenBerg, D. A., Bergbusch, Peter A., Dowler, Patrick D. 2006, ApJS, 162, 375
- van Leeuwen, F. 2007, A&A, 474, 653
- Zwitter, T., et al. 2010, A&A, 522A, 54